Optimal Right and Wrong Way Risk
A methodology review, empirical study and impact analysis from a practitioner standpoint

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Right-way and wrong-way risk modelling has gathered increasing attention in the past few years. A number of models have been proposed. At present, there is no indication in the literature as to which of these proposed models is optimal, and calibration is only loosely touched upon. Also, while existing papers in the area focus in CVA, other very important credit-driven risk metrics such as initial margin, exposure management and regulatory capital can also notably be affected by right-way and wrong-way risk. Here, the authors extend the current state-of-the-art research in right- and wrong-way risk methodologies with a comprehensive empirical analysis of the market-credit dependency structure. They utilise 150 case studies, providing evidence of what is the real market-credit dependency structure, and giving market calibrated model parameters. This study offers the pillars of a stochastic corre-

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lation model, driven by empirical data, that could be optimal for pricing and risk management of complex structures. Next, using these realistic calibrations, they carry out an impact study of right-way and wrong-way risk in real trades, in all relevant asset classes and fundamental trades. This is accomplished by calculating the change in many major credit risk metrics that banks use (CVA, initial margin, exposure measurement, capital) when this risk is taken into account. All this both for collateralised and uncollateralised trades. The results show how these credit metrics can vary quite significantly, both in the “right” and the “wrong” ways. Finally, based on this impact study, the authors explain why a good right and wrong way risk model is central to financial institutions, furthermore describing the consequences of not having one.

The market swings that we saw in 2008 highlighted the importance of counterparty credit risk in over-the-counter (OTC) financial derivative contracts. Accounting frameworks now require CVA to be a part of balance sheet calculations (IFRS 13, FAS 157). In fact, it has been reported that around two thirds of the credit losses in the 2008-09 financial turmoil were CVA losses [6].

When calculating any credit exposure metric, like the Expected Exposure ($EE_t$) or the Potential Future Exposure ($PFE_t$) profiles [1], we need to measure exposure at default. The “at” is a subtle but crucial concept. From a mathematical standpoint, to model the former case we need to build a dependency structure between potential counterparty default events and the portfolio value. When this dependency is non-negligible, it is often said that there exists a “market-credit” dependency.

When this dependency is such that the exposure increases with the probability of default, it is said that we have positive dependency. In these cases, the structure of the trade “exacerbates” the credit risk embedded in it, and hence it is said that we have wrong-way risk (WWR). However, when it works the other way round, that is when the exposure decreases as the default probability increases, it is usually said that we have negative dependency, and this effect is called right-way risk (RWR), since the size of the credit risk decreases as the counterparty approaches a potential default.

For simplification, we shall refer to the joint effect of market-credit dependency that

\[\text{Optimal Right and Wrong Way Risk}\]
creates either RWR and WWR as “directional-way risk” (DWR), as they only are two sides of the same coin.

This DWR effect appears mainly in four asset classes: equity, foreign exchange (FX), commodities and credit. For example, equity prices can be highly dependent to default events of companies in the same region or economic sector, FX rates can suffer major swifts if a government, or a very large company, defaults. The financial performance of some companies, and hence their default probability, can be highly dependent on the price of commodities; e.g. in the energy, mining or transportation sector. Finally, a CDS offering default protection on a given bank, sold or bought by another bank in the same region, will show a high degree of DWR.

To the author’s knowledge, there are the existing fundamental methodologies for DWR modelling.

- Under the Basel framework, RWR is not considered; rather, only WWR is taken into consideration. WWR is accounted for by increasing the exposure metric by a constant factor $\alpha$ across the board, for all counterparties and nettings sets without any particular considerations.

- Cesari et al. [2] and Iacono [5] propose to change the risk measure in the Risk Factor Evolution (RFE)$^2$ models that drive the exposure calculations.

- A ‘brute force approach’ technique would be to simulate a counterparty default flag in sync to market values, and utilise for the exposure calculation only those scenarios where a default has happened. An alternative approach is to simulate a default flag subject to market values, and then weight each scenario by

$$w_{t,i} = \frac{m_{t,i}}{M}$$

where $m_i$ is the number of defaults, and $M$ the number of default simulations.

- Another family of models do an analytical or semi-analytical change of risk measure in the exposure metric calculation. In these models, those weights $w_i$ are estimated with either a Merton model approach (e.g., see Cespedes et al. [3], D. Rosen & D. Saunders [7] and Cesari et al. [2]), an empirical analysis (e.g., see Ruiz [8] and Hull & White [4]), or a portfolio value approach (e.g., see Hull & White [4]).

- A final set of models consist in making a hypothesis regarding the market stress that the markets would suffer subject to a sovereign default, and then further hypothesise the correlation between that event and a company default [9].

All these models have positive and negative characteristics. However, one of the major problems that most of them have is that they cannot be calibrated to any data:

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$^2$By RFE models it is meant all the models of market factors (e.g. interest rates, FX rates, equity prices, commodity prices, implied volatilities, credit, etc) that drive the exposure metrics.
rather, the calibration can only be ‘guessed’. For example, we cannot measure the correlation in a Merton model between the default events and the value of the portfolio of derivatives.

However, there is one model that can actually be calibrated to the market. If we follow the empirical analysis model, then we can infer the dependency structure between the counterparty default events and the value of the portfolio of derivatives with that counterparty from market data. For this reason, the authors think this is the optimal framework for right- and wrong-way risk modelling.

In this piece of work the authors are going to show with real data how a DWR can be implemented, showing the impact that it has in the most important counterparty risk metrics: CVA for pricing, Initial Margin (IM) and Potential Future Exposure (PFE) profiles for risk management, and the regulatory Effective Expected Positive Exposure (EEPE) and regulatory CS01 for capital calculation.

The Model

Let’s develop an empirical model for DWR.

In this model, we want to change the risk measure when calculating the different risk profiles like $EE_t$ or $PFE_t$. If we are at a given time point in the future $t$, let $\Psi_t(P)$ be the non-DWR distribution of prices for the portfolio under study. We want to transform this distribution into another one ($\tilde{\Psi}_t(P)$) that contains DWR information. We can do this via a weight function so that

$$\tilde{\Psi}_t(P) = \Psi_t(P) \ w_t(P) \tag{2}$$

It must be noted that this $w_t(P)$ can be also interpreted as a Radon-Nikodym derivative.

In practice, typically we are going calculate this numerically via a Monte Carlo (MC) simulation, and so if $i$ counts through the scenarios in the simulation,

$$\tilde{\Psi}_{t,i} = \Psi_{t,i} \ w_{t,i} \tag{3}$$

As seen in Equation 1, the weight $w_{t,i}$ can be seen as the default probability\(^3\) of our counterparty at the time point $t$ under the scenario $i$.

Suppose that we find a market factor ‘x’ so that the default probability of a counterparty can be expressed in the form

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\(^3\)Strictly speaking, the default probability during the next simulation time step subject of having survived up to $t$.\n
\[ PD = g(x) + \sigma \epsilon, \]  

(4)

where \( \epsilon \) is a normalised random number that can follow, in principle, any distribution. We are going to refer to the variable ‘\( x \)’ as the DWR driving market factor. This factor could be an equity price, an FX rate, a commodity price, or any market variable in which we observe a relationship as described by Equation 4. If the MC simulation that is used to compute the counterparty credit risk metrics already contains a simulation for \( x \), with a given dependency structure to all other market factors, then we can use those values of \( x \) in each scenario to obtain the necessary information for the default probability for that counterparty in that scenario and time point, and hence of the weight \( w_{t,i} \). In other words, we can say that

\[ w_{t,i} = g(x_{t,i}) + \sigma \epsilon. \]  

(5)

In this framework, the task of the researcher is to discover the best DWR driving factor and optimal functional form for Equation 4.

For example, it has been observed that an equity price could be used as the market factor \( x \), and Equation 4 could be calibrated using the default information embedded in the credit spreads; Ruiz [8] showed with one illustrative example (Ford) how a functional form \( g(x) = Ax^B \) could be an optimal candidate for this purpose.

**Empirical Study**

In this section we are going to study actual data with the aim of finding empirical dependencies that will subsequently deliver a realistic and easily-calibrated (i.e. data-driven) DWR framework. That is, our aim now is to calibrate Equation 4 from empirical data.

One of the major problems of studying historical default events is that they happen quite rarely and hence it can be very difficult to obtain data that is statistically significant. This problem is even more acute when trying to find data on defaults with a DWR component in it. For this reason, in order to obtain relevant information for our purposes, it may be best to use the market available information about default probabilities that is embedded in the CDS prices, which can be daily traded. In particular, we can take advantage of a widely used approximation for the instantaneous default intensity (\( \lambda \)), given by [4]

\[ \lambda = \frac{s}{1 - RR}, \]  

(6)

where \( s \) is the credit spread and \( RR \) is the expected recovery rate on the event of default.
In particular, we will use data from the one-year par credit spread as this tenor should provide a good proxy for the market’s view of the short-term default probability and, most importantly, it is amongst the most liquid tenor points in the CDS market; hence we should be able to find reasonably good quality data.

We now discuss dependency structures for each of the cases previously laid out.

Our final aim is to approximate the best functional form for \( g(x) \) in Equation 4. To this end, we tried four different forms for \( g(x) \); power, exponential, logarithmic and linear.

\[
\begin{align*}
g_1 &= A_1 x^{B_1} \\
g_2 &= A_2 e^{B_2 x} \\
g_3 &= A_3 + B_3 \ln x \\
g_4 &= A_4 + B_4 x
\end{align*}
\]

We used a least-squared liner regression method to estimate \( A \) and \( B \) for each of those functional form \( g_i \), using five years of historical weekly data. To assess the relative quality of each of these fits, we utilised two parameters: the \( R^2 \) delivered by the regression and the size of \( \sigma \), which was obtained as the the standard deviation of \( \{ \lambda_i - g(x_i) \} \). In principle, we are looking for the fit that delivers the highest \( R^2 \) and/or the lowest \( \sigma \).

**Equity**

For this type of transacions, we want to understand the real (i.e. empirically measured) dependency between a corporate equity price and its probability of default \( PD \). The often used Merton model delivers a dependency close to an exponential function, which may or may not be the case with real data. As said, we will obtain this information from the default intensity embedded in its one-year CDS spread.

To do this, we selected a number of representative firms, covering a wide range of regions and activity sectors. The list of 71 equity prices used can be seen in the Appendix.

We give an illustrative example (Societe Generale) of that dependency in Figure 1. The scatter plots of the default probability versus the equity price show a a clear dependency structure between the equity price and the default probability. As expected, data shows that the lower the equity price, the higher the market-expected default probability.

The following table show the values of \( A \), \( B \), \( R^2 \) and \( \sigma^2 \) for the illustrative example (Societe General) as per Equation 7. The calibration results for all equities studied are shown in the Appendix.
We can use two methods to decide which of those fits is best. On one hand, if we measure the quality of the fit by its $R^2$, in 53% of the cases the functional form with the highest $R^2$ was the power law, 27% of the times it was the exponential and 20% times it was the logarithmic. On the other hand, if we want a model that needs the smallest noise in Equation 4, then we should categorise each functional form by the size of $\sigma$, and in this case the power function is the clear winner: in 100% of the cases it delivered the smallest $\sigma$.

To summarize, data shows a clear “inverse” dependency structure an equity price and the default probability of that firm, as somewhat expected. More precisely, a power law could be seen as the best functional for for $g(x)$ as it always delivers a good $R^2$ compared to other fits and it always minimises the noise term $\sigma$ in our data set.

**Foreign Exchange**

One of the key indicators of the performance of an economy is the demand for its currency. For this reason, currency devaluations tend to happen in parallel to deterioration of country-wide economic performance and credit quality, and so we may be able to use an economy’s currency value as the asset ($x$) that drives a DWR model. This can be particularly important for sovereign risk, as governments do not have an “equity price”,
but it can also be applied to firms in emerging economies, as we shall see.

In order to investigate the Foreign Exchange (FX) rate as a DWR driving asset, we studied the dependency between a number of currencies of emerging economies (against USD) and the default probability implied by one-year credit spreads from sovereigns and corporates. The list of sovereign and companies considered are shown in the Appendix with all the calibrations.

We calibrated each of these sets of data to power, exponential, logarithmic and linear fits for \( g(x) \). The results in the case of Brazil government as shown in the following table.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>( R^2 )</th>
<th>( \sigma^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>( 4.179 \cdot 10^{-4} )</td>
<td>5.385</td>
<td>70%</td>
<td>( 3.724 \cdot 10^{-5} )</td>
</tr>
<tr>
<td>Exponential</td>
<td>( 3.368 \cdot 10^{-4} )</td>
<td>1.908</td>
<td>69%</td>
<td>( 4.749 \cdot 10^{-5} )</td>
</tr>
<tr>
<td>Logarithmic</td>
<td>( 7.435 \cdot 10^{-2} )</td>
<td>(-3.129 \cdot 10^{-2})</td>
<td>54%</td>
<td>( 5.649 \cdot 10^{-5} )</td>
</tr>
<tr>
<td>Linear</td>
<td>( 3.984 \cdot 10^{-2} )</td>
<td>(-5.972 \cdot 10^{-2})</td>
<td>57%</td>
<td>( 5.213 \cdot 10^{-5} )</td>
</tr>
</tbody>
</table>

The above numbers show that the quality of fit was generally best for either an exponential or power law.

It must be noted that this methodology cannot be used for those currencies that are undergoing government intervention, as their FX-credit dependency structure is artificially biased.

**Commodities**

Commodities can also act as good DWR driving assets. This is the case for those firms whose business is highly dependent on a given commodity. In some cases, the performances of those companies strengthen when the commodity prices increase (e.g. oil and gas extraction, mining, agricultural), and in some other cases the performance will be at risk when prices increase (e.g. airlines, construction). We should expect this relationship to be revealed in the data.

We performed this analysis using Copper, Aluminium, Gas, Oil and an Oil & Gas index\(^4\) as the asset for \( x \). Appendix shows the list of companies compared with each of the assets with its calibrations.

The following table show the fit results for Xtrata v. copper price.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>( R^2 )</th>
<th>( \sigma^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>( 2.036 \cdot 10^5 )</td>
<td>(-2.780)</td>
<td>92%</td>
<td>( 2.243 \cdot 10^{-4} )</td>
</tr>
<tr>
<td>Exponential</td>
<td>( 7.878 \cdot 10^{-1} )</td>
<td>(-1.159 \cdot 10^{-2})</td>
<td>93%</td>
<td>( 3.731 \cdot 10^{-4} )</td>
</tr>
<tr>
<td>Logarithmic</td>
<td>(-1.606 \cdot 10^{-1})</td>
<td>(9.585 \cdot 10^{-1})</td>
<td>81%</td>
<td>( 5.161 \cdot 10^{-4} )</td>
</tr>
<tr>
<td>Linear</td>
<td>(-5.456 \cdot 10^{-4})</td>
<td>(2.129 \cdot 10^{-1})</td>
<td>69%</td>
<td>( 8.441 \cdot 10^{-4} )</td>
</tr>
</tbody>
</table>

\(^4\)Consisting of the average of the WTI and Gas price.
On this occasion, the data again demonstrates clear dependency structures that reflect the underlying business of each of the firms.

**Credit**

We would also like to mention the case of the credit asset class here, for the sake of completeness, but DWR models in this class are quite trivial, since if we already have a credit model in our system that has a dependency structure with all other asset classes, we may use it to estimate \( w_{t,i} \) in Equation 5 as

\[
   w_{t,i} = \hat{\lambda}_{t,i},
\]

where \( \hat{\lambda}_{t,i} \) represents the model’s short-term default intensity for scenario \( i \) at the simulation time \( t \).

**More Relationships Found**

During the course of this empirical study, we uncovered other interesting data dependencies.

**European Sovereign Risk**

Europe is a special case of sovereign risk, as we cannot use the FX model for the countries that have been under financial stress (Spain, Portugal, Italy, Greece and Ireland). This is because the Euro currency is also being affected by the economic situation of those member states that are net creditors within the Euro area. In fact, we tested this and no clear functional form was found between the USD/EUR exchange rate and the default intensity of those European countries under stress. These results are given in the Appendix.

However, trying to find a way to model DWR for these sovereigns, we noticed a strong dependency between the governments’ credit qualities and the levels of the typical equity indices for each of these countries. The case of Greece is shown in Figure 2 and the following table, and all the results are given in the Appendix.

<table>
<thead>
<tr>
<th>Model</th>
<th>( A )</th>
<th>( B )</th>
<th>( R^2 )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>( 4.944 \cdot 10^5 )</td>
<td>(-1.866 )</td>
<td>57%</td>
<td>( 9.400 \cdot 10^{-1} )</td>
</tr>
<tr>
<td>Exponential</td>
<td>( 2.440 )</td>
<td>(-1.479 \cdot 10^{-4} )</td>
<td>60%</td>
<td>( 1.516 \cdot 10^0 )</td>
</tr>
<tr>
<td>Logarithmic</td>
<td>(-1.509 )</td>
<td>( 1.204 \cdot 10^4 )</td>
<td>45%</td>
<td>( 1.193 \cdot 10^3 )</td>
</tr>
<tr>
<td>Linear</td>
<td>(-5.310 \cdot 10^{-4} )</td>
<td>( 1.910 \cdot 10^0 )</td>
<td>26%</td>
<td>( 1.618 \cdot 10^0 )</td>
</tr>
</tbody>
</table>

This framework seems to provide an excellent way to model DWR with those sovereigns.
Emerging Market Equity

Following up to the above study, we expanded it to emerging economies and explored the link between the country’s main equity index and the default probability of its sovereign debt. The results are shown in the Appendix. We found that there is a clear relationship that can be used to model DWR too, and so this can be an effective alternative to using the FX rate as the DWR driving asset when, for example, the currency free-floating is manipulated.

Conclusions of this Empirical Study

The analysis performed illustrates the adequacy of the Empirical Analysis approach to model DWR. It shows several cases where a clear dependency between the default intensity can be drawn with a given asset. That asset can be equity prices, FX rates or commodity prices depending in the nature of the counterparty under study.

We have seen that, for corporates, the equity price appears to be a good candidate for the DWR driving asset, but not necessarily the only one, as we may be able to use the FX rate or a commodity price as the driving asset too. If more than one is available, it is up to the researcher to decide which is best to use; in fact, a blend of them could be used.

For sovereigns, we have seen that the FX rate of its currency can be a good driving asset. In the special case of European sovereign risk, or when the FX market is undergoing government intervention, we have seen that we may use the country equity index instead.
Regarding the functional form for $g(x)$, we have seen that, from the ones tested, a power-law seems to work quite well in general, followed by an exponential law. However, there is no reason why a practitioner designing a model for a specific counterparty should not investigate other more elaborate forms for $g(x)$.

### The dependency structure between market and credit

The authors would like to point out that all these findings can also be used outside of the DWR scope. The data shown here imply clear market-credit dependency structures between asset classes (equity-credit, FX-credit, commodity-credit, equity-FX-commodity-credit in emerging market firms, etc.) that go beyond the typical linear correlation models, and hence they can (arguably, they should) be used to model those dependencies in a general framework, even outside of the DWR environment.

### Correlation implied by the Empirical Analysis

Let’s say that the default probability of a company is given by

$$\lambda = g(x) + \sigma \epsilon$$  \hspace{1cm} (9)

where $x$ is a market driving factor. A change in that default probability is approximately given by

$$\Delta \lambda = g'(x) \Delta x + \sigma \Delta \epsilon$$  \hspace{1cm} (10)

The correlation between a change in the default probability and a change in the driving factor is given by

$$\rho(x) = \frac{\sigma_{\lambda x}}{\sigma_{\lambda} \sigma_{x}}$$  \hspace{1cm} (11)

where $\sigma_{\lambda x}$ is the covariance between $\Delta \lambda$ and $\Delta x$, and $\sigma_{\lambda}$ and $\sigma_{x}$ are the square root of the variance of $\Delta \lambda$ and $\Delta x$ respectively. Making use of Equation 11 we can see that

$$\rho(x) = \frac{g'(x) \sigma_{x}}{\sqrt{g'(x)^2 \sigma_{x}^2 + \sigma^2}}$$  \hspace{1cm} (12)

It should be noted that this correlation is a function of the market driving factor $x$, which is quite natural result given the data observed. For example, in the case of equities, when the equity price is very high, a change in the equity value tends to influence minimally
its credit standing, hence correlation should be low. On the other hand, when the equity price is very low, a small change in the equity value tends to be highly linked to strong changes in its credit worthiness, hence correlation should be high. This observation is indeed embedded in \( g'(x) \), as seen in the formula obtained.

A stochastic correlation model

All this has major implications in terms of risk modelling. The empirical data, that drives the functional form of \( g(x) \), shows how a constant correlation model may be far from good to describe the observed market-credit correlations.

In fact, this modelling framework builds quite naturally an stochastic correlation model. If we have a stochastic model for the the variable \( x \), we are implicitly having a stochastic model for the correlation via Equation 12. Given that \( g(x) \) is obtained by empirical data, this framework is going to offer, in our view, an optimal way to model the market-credit dependency structures.

This modelling scheme has been investigated in the context of DWR, but it can be used beyond that. For example, price and risk metrics of portfolios of securities (e.g., CDOs and other exotic derivatives, portfolios of derivatives, etc) can be highly driven by dependency structures. Indeed, this is specially true in risk management, as risk metrics that measure tail risk are nearly always quite sensitive to correlations, and so risk management in a financial institution is always interested in the correlation between credit risk and market risk. This “driving market factor approach” might be an appropriate way of proceeding for risk metrics like, for example, initial margin asked by central clearing houses, as they are based in highly conservative risk metrics that are very sensitive to correlations.

Impact of DWR in Counterparty Credit Risk Calculations

We now wish to understand the impact of DWR in the real world. To do this, we run the preferred modelling framework through a number of sample trades (options, forwards and swaps), with and without a DWR model. When modelled with a DWR, it is realistically calibrated to actual market conditions. We do this for all asset classes that show a market-credit dependency structure and that subsequently lead to DWR: equities, FX and commodities. We then study the impact of DWR in the CVA, initial margin, future exposure and regulatory capital calculations.

For CVA, we calculate unidirectional CVA for simplicity. For initial margin, the maximum of the PFE profile at 99% confidence. For exposure management, we use the PFE profile at 90% confidence, and for regulatory capital, both the EEPE and the regulatory
CS01 as defined by the Basel Committee.

It is not the goal of this paper to discuss details of collateral modelling, thus we only concern ourselves with an ideal CSA: daily margining, zero threshold, zero minimum transfer amount, zero rounding, etc, and a close-out period of ten days.

For simplicity, all risk factor evolution models (equity, FX and commodities) operate on one-factor geometric Brownian motion calibrated to the risk-neutral measure as of January 2013. There is no market for DWR trades, so \( g(x) \) cannot be calibrated to a market-implied measures; it will be calibrated using five years of weekly historical data as of January 2013. Also, along with Hull & White [4], we have seen that the noise term \( (\sigma) \) in Equation 4 did not have any noticeable effect in results, so we are disregarding it in the calculations.

A number of simplifications were employed for the RFE diffusion and the derivatives pricers, which were seen to have no notable effect on the calculations.

**Long FX Forward**

Suppose that we sell to Petrobras, a major oil company in Brazil, a one-year FX forward on the USD/BRL exchange rate. Data shows a clear dependency between the FX level and the default intensity of this company best fitted by 

\[
g(x) = 3.1481 \cdot 10^{-4} x^{6.4313},
\]

where \( x \) is the USDBRL exchange rate (see the Appendix). Figures 3 and 4 show the impact of DWR in the counterparty risk metrics, both if the trade is uncollateralised (Figure 3) and collateralised (Figure 4).

In this transaction, we are long the USD/BRL rate. Thus DWR that we have in the uncollateralised case is such that when the BRL devalues, the default probability of Petrobras increases and the forward is in-the-money for us; so we have wrong-way risk. This is clearly shown in the PFE-90% profile in Figure 3. As a result, all CVA, Initial Margin (IM) and regulatory capital increase most notably compared to its non-DWR value.

In the collateralised case, the DWR effect we observe is also wrong-way risk, although smaller this time. This is because (i) the MC paths that carry the most weight \( w \) are those in which USD/BRL is high and (ii) the 10-day changes of the forward, that is a delta-one product, are bigger in those paths with high \( w \) (as a result of the geometric nature of the FX rate moves). As a consequence, we have a wrong-way risk effect. This

5The version where a IMM compliant market risk VaR engine calculates credit VaR based on parallel shifts of the credit curve.

6Also called Margin Period of Risk by in the context of the Basel Committee.

7RFE models are based on geometric Brownian motion with constant volatility. Pricing models are Black-Scholes for options, risk-neutral valuation for FX forwards and the approximation of continuum payments for swaps. A single yield curve per currency was used, assumed to be flat and non-stochastic. Implied volatilities are also assumed to be flat and non-stochastic (as the main driver of risk for Black-Scholes options is the underlying spot price).
Figure 3: Impact of DWR modelling in counterparty credit risk metrics in an uncollateralised long FX forward.

Figure 4: Impact of DWR modelling in counterparty credit risk metrics in a collateralised long FX forward.
effect is small, compared to the uncollateralised case, because exposure is only sensitive to 10-day moves in the FX rate.

**Long Equity Call Option**

Suppose now that we buy a 5-year at-the-money call option from Bank of America, with JP Morgan as the reference entity, and we use the power function that we have obtained for Bank of America, \( g(x) = 0.44749 x^{-1.2216} \), where \( x \) is the stock price of Bank of America. Figures 5 and 6 show the impact of this DWR in this trade for the uncollateralised and collateralised cases respectively.

The uncollateralised case (Figure 5) shows a strong right-way risk behaviour. This is because the paths in the MC simulation that carry highest weight are those where the Bank of America stock prices are low, but in those cases the option will tend to be out-of-the-money, and so the exposure weighted by the counterparty default probability is lower than when the weighting is not considered. The impact is very strong in this case: a reduction of nearly 50% in CVA and IM, a very substantial decrease of the exposure profile and a decrease of around 10% of CCR capital and of around 40% of CVA capital. We would like to remind the reader that these measurements are completely realistic, calibrated as of January 2013.

In the collateralised case (Figure 6) the DWR effect is very strong too, as the MC paths that carry high weight \( w \) (when the stock prices are low) have a small delta. So the
10-day changes in the option price are very small when \( w \) is big. As a result we can see a right-way risk that nearly halves the CVA and IM, decreases the exposure profile quite dramatically, reduces CCR capital by about 10% and CVA capital by about 35%.

**Commodity Payer Swap**

Let’s say that we trade with Canadian Natural Resources a two-year WTI swap in which we receive floating and pay fixed. We have seen in the empirical study that the CDS market of this counterparty links quite clearly the level of the WTI oil spot price to its default probability, and we have seen that a power law seems be best for that dependency, \( g(x) = 2.296 \cdot 10^{-11} x^{-6.811} \) where \( x \) is the WTI price. The counterparty credit risk metrics with and without DWR are shown in figures 7 and 8.

When the trade is uncollateralised (Figure 7), as the price of WTI increases, the swap will become more valuable for us and hence the credit exposure increases, but at the same time the counterparty default probability will decrease, and hence this trades shows right-way risk. This effect is quite dramatic as a result of the strength of the market-credit dependency captured by \( g(x) \): CVA gets divided by around 6, IM more than halves, the exposure profile is strongly reduced and both CCR and CVA capital also decrease most heavily.

When the trade is collateralised, we appear to experience right-way risk. This is because
Figure 7: Impact of DWR modelling in counterparty credit risk metrics in an uncollateralised payer oil swap.

Figure 8: Impact of DWR modelling in counterparty credit risk metrics in a collateralised payer oil swap.
of the two effects that we previously saw in the case of the forward\(^8\), which comes at no
surprise as a swap can be seen as a strip of forwards.

A Remark on this Example

We would like to draw the reader’s attention to this example, as in the authors’ views it demonstrates the strength of the DWR model used.

We are calibrating the Empirical Analysis approach to model DWR to real data as of January 2013. As a result we are easily able to incorporate into our DWR model the *empirically measured* dependency structure between the credit quality of the counterparty and the market factors, as opposed to a “guesstimate” of it, which is the best you can aim for in all other modelling frameworks. As a consequence, we are actually matching the intensity of the DWR effect to that indicated by the market. In this example we observe a very significant reduction in several credit risk metrics when DWR effects are considered. Such reduction would, in our experience working for many years in financial institutions, be very difficult to justify to a regulator, for example, without a data-driven modelling framework like this one. Also, it reduces model risk as it removes subjective judgement by the research team.

Sensitivities

So far we have seen how the main counterparty credit risk metrics (CVA, Initial Margin, PFE exposure and Regulatory Capital) can vary notably in options, forwards and swaps when considering DRW effects. We have reviewed examples where the source of DWR comes from the equity, FX and commodity markets. In addition to this, the *sensitivities* of these metrics can also be importantly affected by DWR. This has major implications regarding optimal CVA hedging strategies.

We have seen already some of this by studying the impact of the DWR model in the regulatory CVA-CS01. To illustrate this case further, we make use of the payer WTI swap from before, and observe how the CVA changes as the WTI volatility varies. We illustrate this here by plotting the CVA price and the CVA vega for different values of the WTI volatility, all else remaining constant. The results are shown in Figure 9, both in an uncollateralised and collateralised basis.

The effect is quite remarkable in both cases. In the uncollateralised case, DWR effects change the Vega most strongly; in fact it even changes in sign. In the collateralised case the is effect also remarkable; the vega with DWR goes to zero quite rapidly as the volatility increases, while it increases in the case without DWR.

This example highlights the point that DWR is important not only for the calculation of counterparty credit risk metrics, but also for its sensitivities. To the author’s knowledge,

\(^8\)A balance between \(g(x)\) and the geometric nature of \(\delta P\).
this type of test has not been done in any other DWR publication, so we cannot be sure about this, but we believe that all other modelling frameworks will miss out this important effect.

Conclusions

What is the optimal way to model DWR? In the authors’ opinion, the Empirical Analysis methodology is optimal for modelling DWR\(^9\). This is because it is the only one that achieve all the following goals.

1. It is based in a robust modelling framework, and it uses the observed market-credit dependency structure, as opposed to a guess of how that dependency structure could be.

2. It can be directly and easily calibrated to data.

3. Its implementation requires minimal work, as it makes uses of existing Monte Carlo simulations without DWR.

\(^9\)Perhaps with the special exception of collateralised FX trades with emerging markets sovereign, where the default itself can trigger a jump in the underlying market and, hence, a scenario-based approach may be needed as explained.
Its impact in the every-day work of a financial institution can be high, as it is based on an intuitive methodology that is data-driven, and it does not make use of latent non-observable abstract variables.

**What is the actual extent of DWR in a financial institution?** We have seen from a number of real examples that DWR can be highly relevant. Our realistic examples calibrated as of January 2013 show that CVA, initial margin, exposure profiles and capital can change significantly; indeed, we have observed a case in which CVA decreased around six times. Those tests had to be done with single trades for illustrative reasons. Each financial institution will have a different book of trades and, hence, it will be difficult to obtain results that can be generalised. However, our results clearly indicate that DWR can be quite important and that it must be dealt with carefully.

**What happens if this DWR is not properly modelled?** The data analysed suggests that the market-credit dependency structure that provokes DWR can be modelled by linking the default probabilities to levels of a market risk factor. We have also seen that the correlation in the changes between those factors and credit drivers is not constant, but in fact depends on the level of the market factor. In our view, a good model for DWR should reflect this.

We have also seen how DWR can impact the credit exposure metrics of a book of trades noticeably, both for collateralised and uncollateralised facilities. Moreover, the drivers of DWR effects are very different in each of those cases.

If a good DWR model is not considered, the counterparty credit risk calculation and risk allocation within an organisation will not adequately reflect the true economic risks that the institution is carrying. As a result, management will not be aware of some actual risks, and it will be bound to make wrong decisions, while incentives will be inappropriately allocated in the organisation and the institution will be exposed to negative events, that are not known to the organisation, and that could have been anticipated and managed pre-emptively with a good DWR model.

A good DWR model will not only reverse those problems, but could also allow a bank to decrease its regulatory capital. This is because, firstly, it may apply to its regulators for a reduction in its α multiplier for capital calculation and, secondly, the EEPE and regulatory CVA-CS01 that drive the capital calculation will be reduced in those netting sets that carry right-way risk. Regulators should be open to these changes as, on this way, they will create the incentives for banks to manage DWR properly. Within the current Basel framework and regulatory policy, risk assessment and allocation is not correct, and incentives to improve the risk management set up regarding DWR are quite non-existent. The authors believe this is an important mistake.
Do we need to consider both right-way and wrong-way risk? We should indeed consider both these factors. Currently, DWR is mainly focused in wrong-way risk in the industry. However, this study shows that right-way is as important as wrong-way for the proper understanding of the economic risks that an institution carries, and so both ought to be considered in parallel.

This modelling framework can (and should) be used beyond DWR Indeed, we have seen how data shows that a constant correlation model for the market-credit dependency structure seems to be quite removed from reality. This should have strong implication in pricing and risk management of complex structures, via either a single trade (e.g., an exotic product) or a number of trades (e.g., a portfolio of OTC derivatives). This modelling framework could be a good way to proceed to calculate risk metrics highly sensitive to correlations.

Acknowledgements

The authors would like to thank Jonathan Campbell, Ersel Korusoy, Christian Morris and the journal’s reviewer for interesting feedback on an early version of this paper.
# Appendix

## Model Calibrations

### Equity calibration:

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**Government FX calibration:**

### Optimal Right and Wrong Way Risk

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### Corporate FX calibration:

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### Commodities calibration:

### European FX Sovereign calibration:

### European Equity Sovereign calibration:
Emerging economy equity v. sovereign calibration:

| Index | Exp | Log | Ln | Pw | Log | Ln | Pw | Log | Ln | Pw | Log | Ln | Pw | Log | Ln | Pw | Log | Ln | Pw | Log | Ln |
|-------|-----|-----|----|----|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Shanghai SE Composite | 6.36-03 | -3.05-05 | -9.16-03 | 7.86-02 | -2.35-02 | 1.36-02 | 98% | 6% | 9% | 5% | 3.06-05 | 4.36-05 | 4.06-05 | 4.16-05 |
| FTSE Bursa Malaysia KLCI | 2.75-04 | -2.85-05 | -4.76-03 | 2.95-02 | -1.35-03 | 5.16-03 | 87% | 84% | 72% | 65% | 3.05-05 | 3.05-05 | 2.95-05 | 2.95-05 |
| KOSPI Index | 1.75-05 | -3.46-05 | -0.96-03 | 8.84-02 | -2.35-02 | 1.45-02 | 87% | 6% | 8% | 5% | 3.06-05 | 4.36-05 | 4.06-05 | 4.16-05 |
| Budapest Stock Exchange Index | 2.03-04 | -2.61-05 | -3.59-03 | 8.76-02 | -2.35-02 | 1.45-02 | 87% | 6% | 8% | 5% | 3.06-05 | 4.36-05 | 4.06-05 | 4.16-05 |
| WSE WIG INDEX | 2.56-06 | -2.61-06 | 0.60-05 | -2.46-03 | -1.35-03 | 1.45-02 | 92% | 8% | 8% | 5% | 3.06-05 | 4.36-05 | 4.06-05 | 4.16-05 |
| NXEX Index | 0.95-09 | -3.11-06 | -3.85-05 | 3.85-04 | -1.35-03 | 1.45-02 | 92% | 8% | 8% | 5% | 3.06-05 | 4.36-05 | 4.06-05 | 4.16-05 |
| KSE National 100 Index | 2.61-06 | -2.61-06 | 1.46-00 | -1.26-03 | -6.81-01 | 1.76-00 | 86% | 6% | 8% | 5% | 3.06-05 | 4.36-05 | 4.06-05 | 4.16-05 |
| Argentina Merval Index | 2.57-06 | -2.61-06 | 1.75-00 | -4.50-03 | -5.46-02 | 0.05-01 | 86% | 8% | 8% | 5% | 3.06-05 | 4.36-05 | 4.06-05 | 4.16-05 |
| Chile Stock Market Select | 2.13-06 | -2.46-06 | 1.38-00 | -2.84-03 | -1.46-02 | 1.76-00 | 86% | 8% | 8% | 5% | 3.06-05 | 4.36-05 | 4.06-05 | 4.16-05 |
| GSCI General Index | 4.50-06 | -2.82-06 | 1.35-01 | -2.02-04 | -4.42-02 | 4.23-01 | 77% | 8% | 8% | 5% | 3.06-05 | 4.36-05 | 4.06-05 | 4.16-05 |
| Mexico IPC Index | 4.35-06 | -2.82-06 | 1.62-01 | -8.52-05 | -5.46-02 | 0.76-01 | 84% | 7% | 8% | 5% | 3.06-05 | 4.36-05 | 4.06-05 | 4.16-05 |
| Tel Aviv 25 Index | 2.05-03 | -1.71-03 | -4.86-02 | -1.45-03 | -2.35-02 | 1.76-01 | 84% | 7% | 8% | 5% | 3.06-05 | 4.36-05 | 4.06-05 | 4.16-05 |
| FTSE/JSE Africa All Share | 3.29-09 | -2.55-06 | 4.35-01 | -2.12-04 | 8.16-01 | 8.55-02 | 89% | 6% | 8% | 5% | 3.06-05 | 4.36-05 | 4.06-05 | 4.16-05 |

References


